Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Graph Theory

Subject Code: 5SC04GRT1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 20/09/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

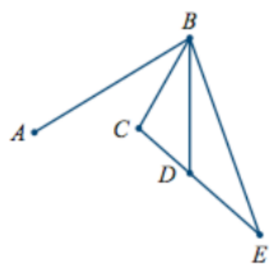
(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a) What is the maximum number of edges in a bipartite graph having 10 vertices? **(02)**

b) What additional arc could be added to the graph to ensure the resulting graph would contain an Euler circuit? **(02)**



c) Define: Arborescence **(02)**

d) True/False: The number of non-zero entries in incidence matrix represents the total number of edges in G. **(01)**

Q-2 Attempt all questions **(14)**

a) State and prove necessary and sufficient condition for the graph is disconnected. **(08)**

b) Define the following: **(04)**
1) Bipartite graph 2) Eccentricity 3) Cut-set 4) Adjacency matrix

c) Define: Circuit matrix **(02)**

OR

Q-2 Attempt all questions **(14)**

a) Let G be a tree with n vertices then prove that G has $n-1$ edges. **(07)**

b) Answer the following questions from the following graph **(05)**

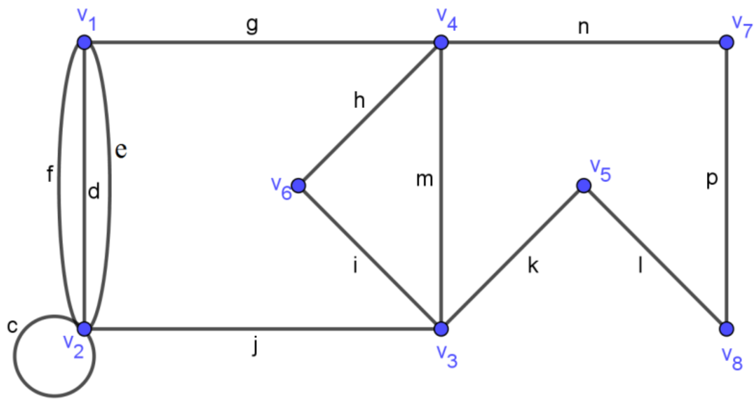


Figure – 1

i) Write one Spanning tree.

ii) Write one fundamental circuit w.r.t. i).

iii) Write adjacency matrix.

iv) Write one closed walk of length 13.

c) Verify first theorem of graph theory for **Figure-1**. **(02)**

Q-3 Attempt all questions **(14)**

a) Let G be a connected digraph with n vertices then the rank of $A(G)$ is $n-1$. **(05)**

b) From the following adjacency matrix draw the diagraph G. Also find X^4 and hence find the directed edge sequence of length four from v_1 to v_3. **(05)**

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

c) Define the following with examples: **(04)**

1) Symmetric digraph 2) condensation

OR

Q-3 Attempt all questions **(14)**

a) Write Teleprinter's problem and hence construct such sequence for $r = 4$ with suitable digraph. **(05)**

b) Define Out-tree. Let G be an out-tree then prove that G is a tree in which every vertex other than root has exactly one in-degree. **(05)**

c) If G be a digraph then prove that determinant of every square sub-matrix of $A(G)$ is 1,-1 or 0. **(04)**

SECTION – II

Q-4 Answer the Following questions: **(07)**

a) Define: Symmetric difference of matching **(02)**

b) What does the mean of chromatic number? Find it for $K_{7,3}$. **(02)**

c) Define: Cut-set matrix. **(02)**

d) Find chromatic number of G if $E \neq \phi$ in any graph G. **(01)**

Q-5 Attempt all questions **(14)**

a) Prove that the vertices of every planner graph can be properly colored with 5 colors. **(07)**

b) Show that the following graphs are isomorphic. **(07)**

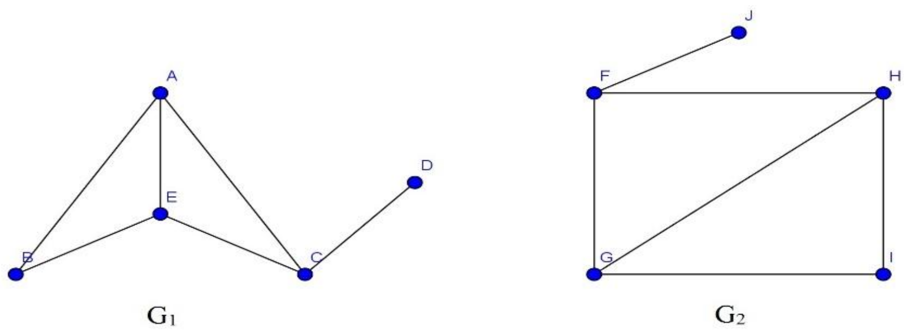


Figure –2

OR

Q-5 Attempt all questions **(14)**

a) Find chromatic polynomial of following graph. **(07)**

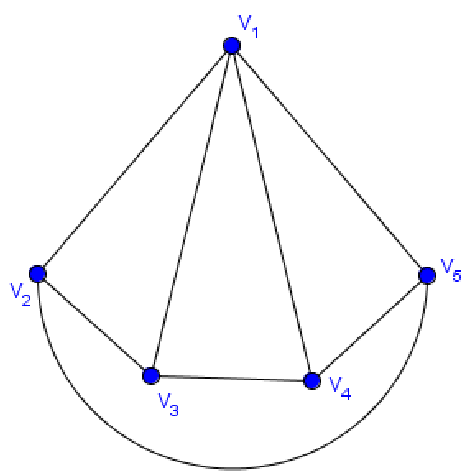


Figure –3

b) Let G be a simple graph with n vertices and $d(v) \geq \frac{n}{2}$, for $\forall v \in G$ then G is a Hamiltonian graph, where $n \geq 3$. **(07)**

Q-6 Attempt all questions **(14)**

a) State and prove Hall's theorem. **(10)**

b) Draw a graph C_5 whose vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$ and edge set $E = \{a, b, c, d, e\}$. Hence find at least two minimum vertex cover and minimum edge cover with definition of each. **(04)**

OR

Q-6 Attempt all Questions **(14)**

a) State and prove Min-Max theorem. **(10)**

b) Answer the following questions from the following graph **(04)**

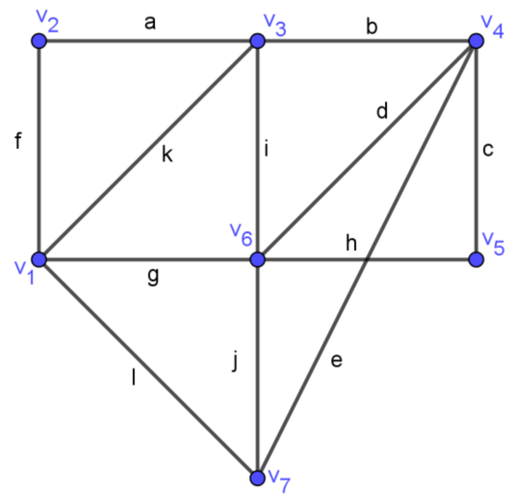


Figure – 4

i) Find a perfect matching and a maximum matching.

ii) Find one M-augmenting path and M-alternating path.